Mathematics
Higher level
Paper 3 – discrete mathematics

Thursday 15 November 2018 (afternoon)

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[50 marks]**.

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Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 8]

(a) Use the Euclidean algorithm to find the greatest common divisor of 77 and 105. [3]

(b) Hence state a condition on c for the Diophantine equation $77x + 105y = c$ to have a solution. [1]

(c) Find all the solutions of the Diophantine equation $11x + 15y = 1$. [4]

2. [Maximum mark: 12]

Let $N = (d_n d_{n-1} \ldots d_1 d_0)_5 = d_n \times 5^n + d_{n-1} \times 5^{n-1} + \ldots + d_1 \times 5 + d_0$ be the representation of a positive integer N in base 5.

Let $a = (143)_5$ and $b = (24)_5$.

(a) Expressing your answers in base 5, calculate

(i) $(a + b)_5$;

(ii) $(ab)_5$. [5]

(b) Show that $(d_n d_{n-1} \ldots d_1 d_0)_5$ is exactly divisible by 4 if and only if $d_n + d_{n-1} + \ldots + d_1 + d_0$ is exactly divisible by 4. [5]

(c) Show that $(x12x)_5$ cannot be exactly divisible by 4. [2]

3. [Maximum mark: 13]

A contagious virus affects the population of a small town with 5000 inhabitants.
Let I_n denote the total number of people who have been infected by the end of the n^{th} week.
In the first week there were 10 cases of infection and by the end of the second week there was a total of 22 cases. A proposed model is that the number of cases is increasing in such a way that the number of new cases in any week is 1.2 times the number of new cases in the previous week.

(a) Show that I_n satisfies the recurrence relation $I_{n+2} - 2.2I_{n+1} + 1.2I_n = 0$. [2]

(b) State appropriate initial conditions. [1]

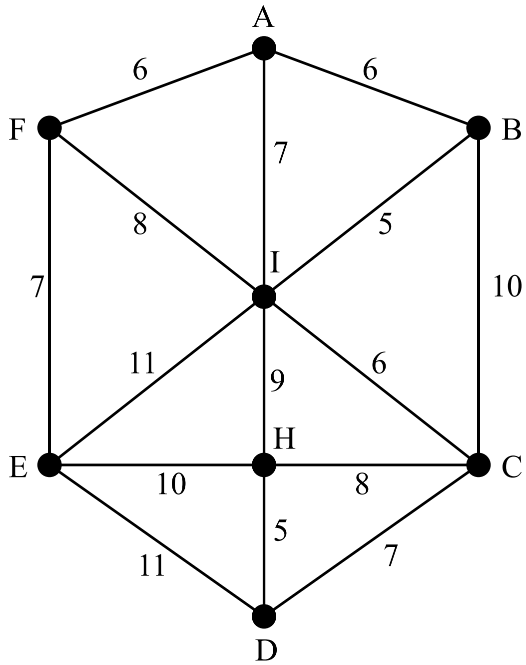
(c) Solve the recurrence relation to obtain an expression for I_n in terms of n. [6]

(d) Hence find during which week the whole town will become infected. [2]

(e) State two limitations of the model. [2]

4. [Maximum mark: 17]

Consider the graph G represented in the following diagram.



(a) State, with a reason, whether or not G has an Eulerian circuit. [1]

(b) Use Kruskal's algorithm to find a minimum spanning tree for G, stating its total weight. Indicate clearly the order in which the edges are added. [4]

The graph G is a plan of a holiday resort where each vertex represents a villa and the edges represent the roads between villas. The weights of the edges are the times, in minutes, Mr José, the security guard, takes to walk along each of the roads. Mr José is based at villa A.

(c) Use a suitable algorithm to show that the minimum time in which Mr José can get from A to E is 13 minutes. [5]

(d) Find the minimum time it takes Mr José to patrol the resort if he has to walk along every road at least once, starting and ending at A. State clearly which roads need to be repeated. [7]